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Control of atomic entanglement by dynamic Stark effect

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We study the entanglement properties of two three-level Rydberg atoms passing through a single-mode cavity. The interaction of an atom with the cavity field allows the atom to make a transition from the upper most (lower most) to the lower most (upper most) level by emission (absoprtion) of two photons via the middle level. We employ an effective Hamiltonian that describes the system with a Stark shifted two-photon atomic transition. We compute the entanglement of formation of the joint two-atom state as a function of Rabi angle gt. It is shown that the Stark shift can be used to enhance the magnitude of atomic entanglement over that obtained in the resonant condition for certain parameter values. We find that though the two-atom entanglement generally diminishes with the increase of the two-photon detuning and the Stark shift, it is possible to sustain the entanglement over a range of interaction times by making the detuning and the Stark shift compensate each other. Similar characteristics are obtained for a thermal state cavity field too.

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I. INTRODUCTION

The most interesting idea associated with composite quantum systems is quantum entanglement. A pair of particles is said to be entangled in quantum mechanics if its state cannot be expressed as a product of the states of its individual constituents. Einstein, Podolsky and Rosen[1] were the first to point out certain nontrivial consequences of entanglement on the ontology of quantum theory. The preparation and manipulation of these entangled states lead to a better understanding of basic quantum phenomena. For example, complex entangled states, such as the Greenberger, Horne and Zeilinger[2] triplets of particles are used for tests of quantum nonlocality[3]. Beyond these fundamental aspects, entanglement has become a fundamental resource in quantum information processing, and there has been a rapid development of this subject in recent years[4].

Cavity-QED has been a favourite tool to test the foundations of quantum mechanics including entanglement. Many beautiful experiments have been carried out, and in recent years, entangled states have been created and verified[5]. Maximally entangled states between two modes in a single cavity have been generated using a Rydberg atom coherently interacting with each mode in turn[6]. Practical realization of various features of quantum entanglement are obtained in atom-photon interactions in optical and microwave cavities. Several studies have been performed to quantify the entanglement generated in atom-photon interactions in cavities[7, 8, 9, 10].

The above cavity-QED related investigations involved mostly the absorption or emission of a single photon in an atomic transition. However, involvement of more than one photon, in particular, two photons in the transition between two atomic levels via a non-resonant intermediate level has been known for a long time [11]. The output radiation from such interactions exhibits novel non-classical properties such as sub-Poissonian photon statistics. Needless to say, the idea of squeezed light has originated from a two-photon process[12]. Two-photon processes have also been studied in cavity-QED [13, 14, 15, 16]. It showed compact and regular quantum revivals in the atomic population in the single atom two-photon cavity-QED [13, 14]. Haroche and co-workers have demonstrated experimentally the two-photon maser action in a micromaser cavity[15]. In total, the two-photon process mostly exhibits non-classical properties compared to the one-photon process[16, 17]. Thus, it would be interesting to study the properties of atom-atom entanglement in the framework of a two-photon process.

The two-photon atomic transition process also introduces a dynamic Stark shift in the atomic transition which is related to the magnitude of the electric field of the radiation inside the cavity. This non-trivial effect which is naturally present in actual experiments involving two-photon transitions has to be properly acounted for in the its theoretical analysis, or example in the two-photon micromaser[15]. Various possibilities of exploiting the Stark effect in quantum optical applications have been noticed in recent years. To name a few, schemes for applying the dc as well as the ac Stark shifts towards implementation of quantum logic gates and algorithms[18], and in the improvement of photon sources for interferometry[19] have been suggested. It is thus tempting to study if the Stark shift can be utilized to enhance atom-atom entanglement, as well.

The purpose of the present paper is to investigate the possibility of controlling atomic entanglement by the Stark shift generated in atomic transitions inside cavities. In section II, we derive an effective Hamiltonian which efficiently describes the atom-field two-photon in-

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teraction. This Hamiltonian is quadratic in the field operators, which is at the root of nonclasssical properties that this process exhibits. The properties of the atom-atom entanglement generated through this interaction is studied in section III. We find that the atomic entanglement can be generally sustained, and for certain interaction times enhanced too, by making the Stark shift compensate for the two-photon detuning. In section IV we show that similar trends are also obtained for a thermal cavity field. We conclude the paper with a summary of our results in section V.

II. DERIVATION OF THE EFFECTIVE HAMILTONIAN

We consider a ladder system of a three-level Rydberg atom interacting with a single mode of a microwave cavity field. The middle level may be a group of closely spaced levels removed far away from one-photon resonance. Thus the interaction involves simultaneous absorption (or emission) of two photons between the two atomic levels via a group of (or one) intermediate levels. This is called a degenerate process since the two photons are from the same mode of the radiation field. Let us label the lower and the the upper level as $|g\rangle$ and $|e\rangle$ respectively and the intermediate levels are labelled as $\{|i\rangle\}$ (see Figure 1).

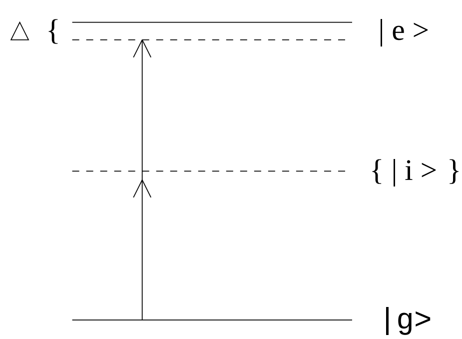

FIG. 1: A three-level Rydberg atom with its three energy levels are denoted by $|e\rangle$ (upper level), $|i\rangle$ (middle level), $|g\rangle$ (lower level) respectively.

The atom initially in the lower state $|g\rangle$ absorbs a photon and jumps to one of the intermediate levels $\{|i\rangle\}$ and from there moves to the upper level $|e\rangle$ by absorbing another photon. The return path of the atom from $|e\rangle$ to $|g\rangle$ is again via $\{|i\rangle\}$ by the emission of two photons to the same mode. To make the equations look simpler let us consider only one intermediate level $|i\rangle$. A microscopically correct Hamiltonian describing the above process can be written as

$$H = H_0 + H_1 \tag{1}$$

where the interaction Hamiltonian H_1 is of the order of one-photon dipole interaction strength. The H_0 and H_1 can be written in operator form as

$$H_0 = \omega_e|e\rangle\langle e| + \omega_i|i\rangle\langle i| + \omega_g|g\rangle\langle g| + \omega a^\dagger a \tag{2}$$

and

$$H_1 = g_1(S_{gi}^{+}a + S_{gi}^{-}a^\dagger) + g_2(S_{ei}^{+}a + S_{ei}^{-}a^\dagger) \tag{3}$$

respectively. g_1 and g_2 are coupling constants (in dipole approximation) for the one-photon interactions responsible for the transitions between the level $|g\rangle$ and $|i\rangle$ and that between $|e\rangle$ and $|i\rangle$ respectively. The atomic operators are given by $S_{gi}^{+} = |i\rangle\langle g|$ and $S_{gi}^{-} = |g\rangle\langle i|$ and similar definitions go for the operators involving the upper level $|e\rangle$. In the basis of the states $|e, n\rangle$, $|i, n+1\rangle$ and $|g, n+2\rangle$, allowed by the rotating-wave approximation, we can write the Hamiltonian in the form of a 3×3 matrix. The density matrix of the atom-field system obeys the equation

$$i\dot{\rho} = [H, \rho]. \tag{4}$$

The equation of motion can be solved by one of the usual known methods, but, the derivations get more and more tedious as the number of intermediate levels increases. However, as these levels are removed far away from one-photon resonance, we can use this to obtain an effective Hamiltonian which efficiently describes the two-photon process. In doing so, we follow the method outlined in Refs.[14, 20]. We start by making a canonical transformation [14, 20] $\rho_r = e^{-iK}\rho e^{iK}$ where K is time-independent and Hermitian. ρ_r obeys the equation of motion

$$i\dot{\rho}_r = [H_{eff}, \rho_r] \tag{5}$$

where

$$\begin{aligned} H_{eff} &= e^{-iK}He^{iK} \\ &= H - i[K, H] - 1/2[K, [K, H]] + ... \end{aligned} \tag{6}$$

We know that the probability of one-photon transition is inversly proportional to the one photon detuning $\omega_{ki} - \omega$ ($k \equiv e, g$). Since this detuning is large, the one-photon transition probabilities are very small. In this situation, it is safe to retain terms upto second order in one-photon coupling constants. Then, retaining terms of the order of square of the coupling constants in the above expression, we have

$$H_{eff} = H_0 + H_1 - i[K, H_1] - \frac{1}{2}[K, [K, H_0]]. \tag{7}$$

Since K is arbitrary, we choose that

$$[K, H_0] = -iH_1, \tag{8}$$

from which we determine all the elements of K. This reduces the effective Hamiltonian to

$$H_{eff} = H_0 - i/2[K, H_1]. \tag{9}$$

Written back in operator form, H_{eff} takes the form

$$\begin{aligned} H_{eff} &= [\Delta + (\beta_e + \beta_g)a^\dagger a]S_z + \frac{1}{2}(\beta_e - \beta_g)a^\dagger a \\ &+ G(S^+ a^2 + S^- {a^\dagger}^2) \end{aligned} \quad (10)$$

where the spin operators are $S_z = \frac{|e\rangle\langle e| - |g\rangle\langle g|}{2}$, $S^+ = |e\rangle\langle g|$ and $S^- = |g\rangle\langle e|$. Δ is the two-photon detuning and is given by $\Delta = \omega_e - \omega_g - 2\omega$ and G is the two-photon coupling constant having the form

$$G = \sum_i \frac{g_1 g_2}{2}\sqrt{(n+1)(n+2)}[\frac{1}{\omega_{ei} - \omega} - \frac{1}{\omega_{ig} - \omega}] \quad (11)$$

where $\omega_{ei} = \omega_e - \omega_i$ and $\omega_{ig} = \omega_i - \omega_g$. The Stark shifts associated with the levels e and g are, respectively,

$$\beta_e = \sum_i \frac{{g_2}^2}{\omega_{ei} - \omega} \quad (12)$$

and

$$\beta_g = \sum_i \frac{{g_1}^2}{\omega_{ig} - \omega}. \quad (13)$$

We now have a Hamiltonian describing the interaction between an an effective two-level system or an effective spin-1/2 system and a single mode radiation field of frequency ω. If we take $\beta_e = \beta_g = \beta$ the Hamiltonian reduces to

$$H_{eff} = [\Delta + 2\beta a^\dagger a]S_z + G(S^+ a^2 + S^- {a^\dagger}^2) \quad (14)$$

It may be noted here that H_{eff} is a function of the two-photon detuning Δ which is an outcome of the procedure followed here. In other words, we need not assume the resonance condition $\Delta = 0$ unlike in other methods in literature[13], but this method gives the two-photon detuning Δ as an independent parameter for the analysis. The effective Hamiltonian can easily be contrasted when compared with a Hamiltonian describing one-photon process [17]. First, the effective Hamiltonian is now cavity photon number dependent and is thus dynamic. Secondly, the Hamiltonian is quadratic in annihilation and creation operators. As mentioned earlier, they are at the root of all the nonclassical behaviours in two-photon processes. Using such a Hamiltonian, we now study the entanglement of two such atoms passing through the cavity one after the other.

III. TWO-ATOM ENTANGLEMENT

The effective Hamiltonian derived above can be written in the matrix form in the basis of $|e, n>$, $|g, n+2>$ states as

$$H_{eff} = \begin{pmatrix} (\frac{\Delta}{2} + \beta n) & g\sqrt{(n+1)(n+2)} \\ g\sqrt{(n+1)(n+2)} & -(\frac{\Delta}{2} + \beta n + 2\beta) \end{pmatrix}. \quad (15)$$

The eigenvalues of H_{eff} are

$$\lambda_1 = -\beta + \sqrt{[\frac{\Delta}{2} + \beta(n+1)]^2 + g^2(n+1)(n+2)}, \quad (16)$$

$$\lambda_2 = -\beta - \sqrt{[\frac{\Delta}{2} + \beta(n+1)]^2 + g^2(n+1)(n+2)}. \quad (17)$$

The corresponding eigenstates can be written as

$$|\Psi\rangle_{\lambda_1} = c_1|e, n\rangle + c_2|g, n+2\rangle, \quad (18)$$

$$|\Psi\rangle_{\lambda_2} = c_2|e, n\rangle - c_1|g, n+2\rangle, \quad (19)$$

where

$$c_1 = \frac{\lambda_1 + (\frac{\Delta}{2} + \beta n + 2\beta)}{\sqrt{g^2(n+1)(n+2) + (\lambda_1 + (\frac{\Delta}{2} + \beta n + 2\beta))^2}} \quad (20)$$

and

$$c_2 = \frac{g\sqrt{(n+1)(n+2)}}{\sqrt{g^2(n+1)(n+2) + (\lambda_1 + (\frac{\Delta}{2} + \beta n + 2\beta))^2}}. \quad (21)$$

We envisage a process in which two atoms pass through an ideal cavity ($Q = \infty$) such that there is no overlap of their flights there [9, 10]. The interaction of each atom in the cavity is described by H_{eff} in Eq.(15). We assume that the two atoms are in their respective upper states $|e\rangle$ before they enter the cavity empty of photons. After passage of the first atom through the cavity, the joint atom-field state is given at any time t is

$$|\Psi_1\rangle_t = (r_1 - is_1)|e, n\rangle + (r_2 - is_2)|g, n+2\rangle. \quad (22)$$

where

$$\begin{aligned} r_1 &= [c_1^2 \cos(\lambda_1 t) + c_2^2 \cos(\lambda_2 t)], & (23)\\ s_1 &= [c_1^2 \sin(\lambda_1 t) + c_2^2 \sin(\lambda_2 t)], & (24)\\ r_2 &= c_1 c_2[\cos(\lambda_1 t) - \cos(\lambda_2 t)], & (25)\\ s_2 &= c_1 c_2[\sin(\lambda_1 t) - \sin(\lambda_2 t)]. & (26) \end{aligned}$$

The second atom then interacts with the cavity field modified by the passage of the first atom. Assuming the flight time of the two atoms through the cavity to be the same, the joint state of the two atoms and the cavity after the second leaves the cavity is given by

$$\begin{aligned} |\Psi_{12}\rangle_t &= (r_1 - is_1)^2|e_1, e_2, n\rangle \\ &+ (r_1 - is_1)(r_2 - is_2)|e_1, g_2, n+2\rangle \\ &+ (r_2 - is_2)(r_1' - is_1')|g_1, e_2, n+2\rangle \\ &+ (r_2 - is_2)(r_2' - is_2')|g_1, g_2, n+4\rangle, \end{aligned} \quad (27)$$

where $r_1' = r_1^{n=n+2}$, $s_1' = s_1^{n=n+2}$, $r_2' = r_2^{n=n+2}$ and $s_2' = s_2^{n=n+2}$. Next we calculate the two-atom mixed state taking trace over the field variables. The joint two-atom mixed state density matrix in the basis of $|e_1, e_2>$, $|e_1, g_2>$, $|g_1, e_2>$ and $|g_1, g_2>$ states is given by

$$\rho_{12} = \begin{pmatrix} \alpha & 0 & 0 & 0 \\ 0 & \gamma & \epsilon & 0 \\ 0 & \epsilon^* & \delta & 0 \\ 0 & 0 & 0 & \eta \end{pmatrix}. \quad (28)$$

where

$$\alpha = (r_1^2 + s_1^2)^2,$$

$$\gamma = (r_1^2 + s_1^2)(r_2^2 + s_2^2),$$

$$\delta = (r_2^2 + s_2^2)({r'_1}^2 + {s'_1}^2),$$

$$\eta = (r_2^2 + s_2^2)({r'_2}^2 + {s'_2}^2),$$

and

$$\epsilon = (r_2^2 + s_2^2)(r_1 - is_1)(r'_1 + is'_1).$$

We compute the two-atom entanglement using the well-known measure of the entanglement of formation[21] given by

$$E_F(\rho) = h\left(\frac{1 + \sqrt{1 - C^2(\rho)}}{2}\right), \qquad (29)$$

where C is called the concurrence defined by the formula

$$C(\rho) = \max(0, \sqrt{\lambda_1} - \sqrt{\lambda_2} - \sqrt{\lambda_3} - \sqrt{\lambda_4}) \qquad (30)$$

where the λ_i are the eigenvalues of $\rho(\sigma_y \otimes \sigma_y)\rho^*(\sigma_y \otimes \sigma_y)$ in descending order, and

$$h(x) = -x\log_2 x - (1-x)\log_2(1-x) \qquad (31)$$

is the binary entropy function. The entanglement of formation is monotone of concurrence. The eigenvalues of $\rho_{12}(\sigma_y \otimes \sigma_y)\rho_{12}^*(\sigma_y \otimes \sigma_y)$ in this case are given by $\alpha\beta$, $\alpha\beta$, $(\sqrt{\gamma\delta} + |\epsilon|)^2$ and $(\sqrt{\gamma\delta} - |\epsilon|)^2$ respectively.

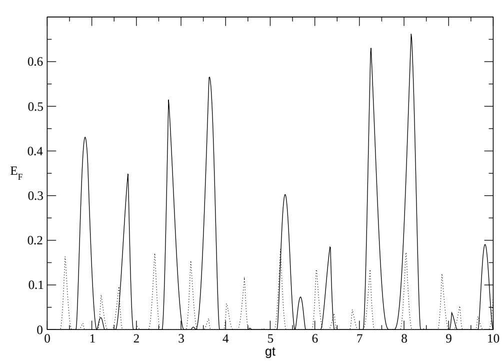

FIG. 2: E_F is plotted vs Rabi angle gt for (i) $\Delta/g = 0$ and $\beta/g = 0$ (solid line) (ii) $\Delta/g = 2$ and $\beta/g = 2$ (dotted line).

We compute numerically the entanglement of formation E_F for the two atoms, and plot it versus the Rabi angle gt for different combinations of the two-photon detuning Δ and the Stark shift β in the Figures 2 and 3. We find that entanglement between the two atoms is controlled by the two-photon detuning and the Stark-shift parameters. We first plot E_F in Fig.2 for the resonant condition ($\Delta = 0$ and $\beta = 0$). Now, if one includes a non-vanishing detuning and the resultant Stark shift, one sees from Fig.2 that the two-atom entanglement is diminished in general over a range of values of the Rabi angle. It can be verified that as long as Δ and β are of the same sign, the magnitude of entanglement between the two atoms decreases with the increase of Δ or β.

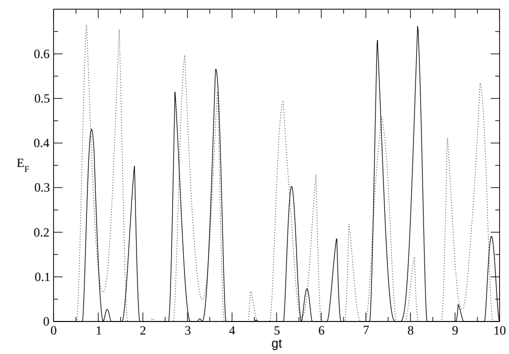

FIG. 3: E_F is plotted vs Rabi angle gt for (i) $\Delta/g = 0$ and $\beta/g = 0$ (solid line) (ii) $\Delta/g = -1$ and $\beta/g = 1$ (dotted line).

The more interesting case is obtained when the detuning and the Stark shifts are of opposite signs. This is revealed in Fig.3 where the two-atom entanglement E_F versus gt for $\Delta = \beta = 0$ is compared with E_F when $\beta = -\Delta = 1$. We see that the magnitude of the entanglement between the two atoms increases when $\beta = -\Delta = 1$ with respect to the case when $\Delta = \beta = 0$ for certain values of the Rabi angle, e.g., $gt \approx 1$. Such enhancement of entanglement is observed in varying measures for other values of the Rabi angle also. Its origin lies in the presence of the photon number operator $a^\dagger a$ in the effective Hamiltonian in Eq.(14) which makes the "effective" two-photon detuning $\Delta + 2\beta a^\dagger a$ dynamic as the cavity photon number oscillates in time. One sees over a range of interaction times that the atomic entanglement is in general sustained with variations in oscillatory behaviour with respect to gt, if the two-photon detuning Δ is compensated by the Stark shift. Or in other words, the maximum entanglement that can be obtained by varying gt over a range of interaction times remains similar to that in the resonant case. This is in striking contrast to the case displayed in Fig.2 where the entanglement reduces substantially with the increase of Δ and β, both having the same sign. It is interesting to note that the entanglement between two atoms is also preserved if we interchange the sign of Δ and β but with the condition $\Delta + \beta = 0$. Overall, we find that the Stark shift acts as a control parameter for the atom-atom entanglement.

IV. ATOMIC ENTANGLEMENT MEDIATED BY THE THERMAL FIELD

The thermal field is the most easily available radiation field, and so, its influence on the entanglement of spins is of much interest. The atomic entanglement mediated by the thermal field through single photon processes have been studied earlier[8, 10]. Since the thermal field is related to the temperature of the medium, photons of this field are naturally present inside the cavity. So it is not out of place to include the Bose statistics for the thermal field in our analysis. The field at thermal equilibrium obeying Bose-Einstein statistics has an average photon number at temperature T^0K, given by

$$< n >= \frac{1}{e^{\hbar\omega/kT} - 1}. \tag{32}$$

The photon statistics is governed by the distribution P_n given by

$$P_n = \frac{< n >^n}{(1+ < n >)^{n+1}}. \tag{33}$$

This distribution function always peaks at zero, i.e., $n_{peak} = 0$. For a field in a thermal state, the joint two-atom-cavity state is obtained by summing over all n, and is given by

$$\begin{aligned} |\Psi_{12}\rangle_t &= \sum_n A_n[(r_1 - is_1)^2|e_1, e_2, n\rangle \\ &+ (r_1 - is_1)(r_2 - is_2)|e_1, g_2, n + 2\rangle \\ &+ (r_2 - is_2)(r_1' - is_1')|g_1, e_2, n + 2\rangle \\ &+ (r_2 - is_2)(r_2' - is_2')|g_1, g_2, n + 4\rangle], \end{aligned} \tag{34}$$

where $P_n = |A_n|^2$ is the photon distribution function of the thermal field. The reduced mixed density matrix of two atoms after passing through the the thermal cavity field in the basis of $|e_1, e_2 >$, $|e_1, g_2 >$, $|g_1, e_2 >$ and $|g_1, g_2 >$ states is given by

$$\rho_{12} = \begin{pmatrix} \alpha_1 & 0 & 0 & 0 \\ 0 & \gamma_1 & \epsilon_1 & 0 \\ 0 & \epsilon_1^* & \delta_1 & 0 \\ 0 & 0 & 0 & \eta_1 \end{pmatrix}. \tag{35}$$

where

$$\alpha_1 = \sum_n P_n(r_1^2 + s_1^2)^2,$$

$$\gamma_1 = \sum_n P_n(r_1^2 + s_1^2)(r_2^2 + s_2^2),$$

$$\delta_1 = \sum_n P_n(r_2^2 + s_2^2)({r_1'}^2 + {s_1'}^2),$$

$$\eta_1 = \sum_n P_n(r_2^2 + s_2^2)({r_2'}^2 + {s_2'}^2)$$

and

$$\epsilon_1 = \sum_n P_n(r_2^2 + s_2^2)(r_1 - is_1)(r_1' + is_1').$$

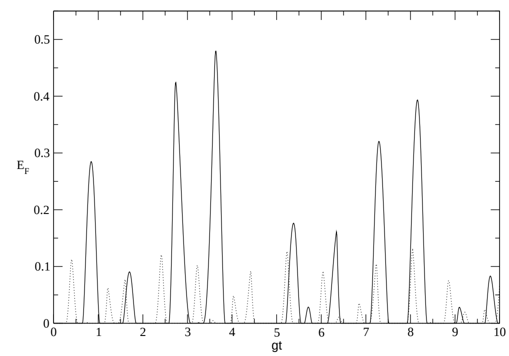

FIG. 4: E_F is plotted vs Rabi angle gt for (i) $\Delta/g = 0$ and $\beta/g = 0$ (solid line) (ii) $\Delta/g = 2$ and $\beta/g = 2$ (dotted line). The average thermal photon number $< n >= 0.1$.

We again compute the entanglement of formation of the joint two-atom state after it emerges from the cavity. Similar to the case of the vacuum cavity field considered in Section III, the thermal field mediates entanglement between the two atoms even though there is no direct interaction between them. This feature was also observed earlier in context of the one-photon atomic transition process[8, 10]. The variation of the magnitude of the two-photon entanglement versus the Rabi angle is displayed in the Figs. 4, 5 and 6.

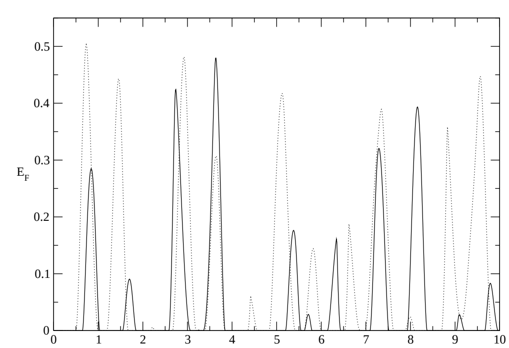

FIG. 5: E_F is plotted vs Rabi angle gt for (i) $\Delta/g = 0$ and $\beta/g = 0$ (solid line) (ii) $\Delta/g = -1$ and $\beta/g = 1$ (dotted line). The average thermal photon number $< n >= 0.1$.

Our purpose here is to investigate the effect of Stark shift on atomic entanglement, and to this end we plot in Figs. 4 and 5 the entanglement of formation as a

function of the Rabi angle gt for different combinations of the two-photon detuning Δ and the Stark shift parameter β when the thermal field has an average photon number $< n >= 0.1$. We notice that the variation in E_F as a function of gt is similar (barring differences in magnitudes) to the case of vacuum cavity field. From Fig.4, we note again, that as long as Δ and β are of same sign, the magnitude of entanglement between the two atoms decreases compared to the resonant case with the increase of Δ or β. But, as seen from Fig.5, atomic entanglement can be increased for particular values of gt by choosing β to be of opposite sign as Δ. Again, similar to the case of the vacuum cavity field, we find that if the two-photon detuning is compensated by the Stark shift (Fig. 5), the atomic entanglement can be sustained on average over a range of values of the Rabi angle. These characteristics are still noticed for higher values of the average thermal photon number $< n >$. As seen from Fig.6, only the magnitude of E_F is reduced with increase of average thermal photons. Thus, the Stark shift can be used to control the atomic entanglement mediated by the thermal field by preserving the maximum magnitude of entanglement obtained in a large range of atom-cavity interaction times.

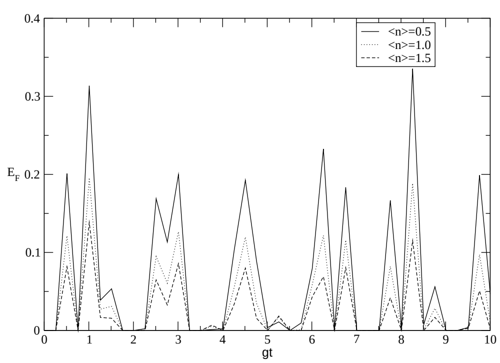

FIG. 6: E_F is plotted vs Rabi angle gt for $\Delta/g = -2$ and $\beta/g = 2$.

V. CONCLUSIONS

In this paper we have investigated the possibility of the control of atomic entanglement by the Stark shift generated in atomic transitions inside cavities. To this end we have considered a degenerate two-photon process in a ladder system and obtained an effective Hamiltonian which describes the interaction efficiently[14, 20]. The two-atom entanglement is shown to be mediated by the cavity field through which the two atoms pass successively without any spatial overlap between them[9, 10]. We are able to use the two-photon detuning which comes out naturally in the method presented in Section II, to compensate the dynamic Stark shift to get the atom-atom entanglement. Through this method we have shown the possibility of using the dynamical Stark shift in controlling atomic entanglement mediated both by the vacuum cavity field as well as by the thermal cavity field.

We have shown that the entanglement between two atoms depends on the two-photon detuning and the Stark shift parameter. The magnitude of atomic entanglement quantified by the entanglement of formation diminishes with the increase of the detuning and the stark shift. However, interestingly, we have found that such a trend could be reversed if the values of the detuning and the Stark shift are made to compensate each other. In the latter case the entanglement could be even enhanced compared to the resonant situation for particular values of the atom-photon interaction time. More generally, it has been shown that the maximum magnitude of entanglement generated over a range of values of the Rabi angle is nearly sustained if we set the values of the two-photon detuning and the stark-shift to be equal and opposite in sign. The effects of photon statistics of the thermal field on the mediated entanglement[8, 10] has also been studied. We have shown that the general characteristics of the atomic entanglement as a function of the photon detuning and the stark shift parameter are maintained for the case of the thermal field inside the cavity. The use of Stark shifts in some quantum information protocols has been suggested recently[18, 19], and an experiment to demonstrate the enhancement of Rydberg atom interactions has actually been perfomed[22]. Our present study should motivate further investigations on the feasability of using Stark shifts in the practical manipulation of quantum entanglement.

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